Engineering Software

P.O. Box 2134
Kensington, MD 20891
Phone: (301) 919-9670
E-Mail: info@engineering-4e.com
http://www.engineering-4e.com

Power Cycles Efficiency Derivation

Here engineering students and professionals get familiar with the ideal simple and basic power cycles and their T - s and p - V diagrams, operation, cycle efficiency derivation and major performance trends when air is considered as the working fluid.

Performance Objectives:

Introduce basic energy conversion engineering assumptions and equations

Know basic elements of Carnot Cycle, Brayton Cycle, Otto Cycle and Diesel Cycle and their T - s and p - V diagrams

Be familiar with Carnot Cycle, Brayton Cycle, Otto Cycle and Diesel Cycle operation

Provide cycle efficiency derivation for Carnot Cycle, Brayton Cycle, Otto Cycle and Diesel Cycle

Understand general Carnot Cycle, Brayton Cycle, Otto Cycle and Diesel Cycle performance trends

Engineering Assumptions

The energy conversion analysis presented in this webinar considers ideal (isentropic) operation and air, argon, helium and nitrogen are considered as the working fluid. Furthermore, the following assumptions are valid:

Power Cycles
Single species consideration -- fuel mass flow rate is ignored and its impact on the properties of the working fluid
Basic equations hold (continuity, momentum and energy equations)
Specific heat is constant

Power Cycle Components/Processes
Single species consideration
Basic equations hold (continuity, momentum and energy equations)
Specific heat is constant

Basic Engineering Equations

Basic Conservation Equations

Continuity Equation

$m = \rho v A$ [kg/s]

Momentum Equation

$F = (vm + pA)_{out - in}$ [N]

Energy Equation

$Q - W = ((h + v^2/2 + gh)m)_{out - in}$ [kW]

Basic Engineering Equations

Ideal Gas State Equation

$pv = RT$ [kJ/kg]

Perfect Gas

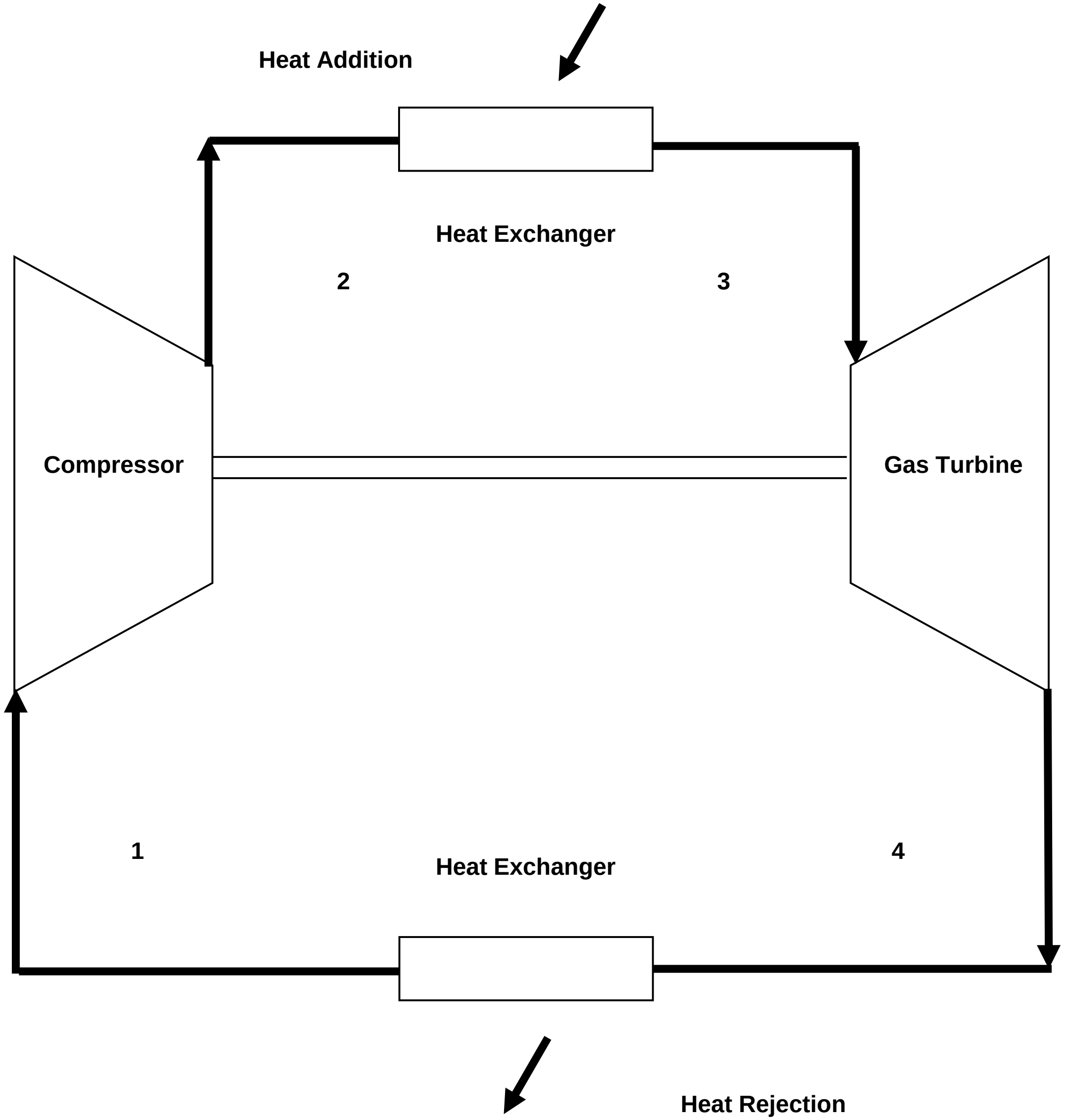
c_p = constant [kJ/kg*K]

Kappa

$\chi = c_p/c_v$ [/]

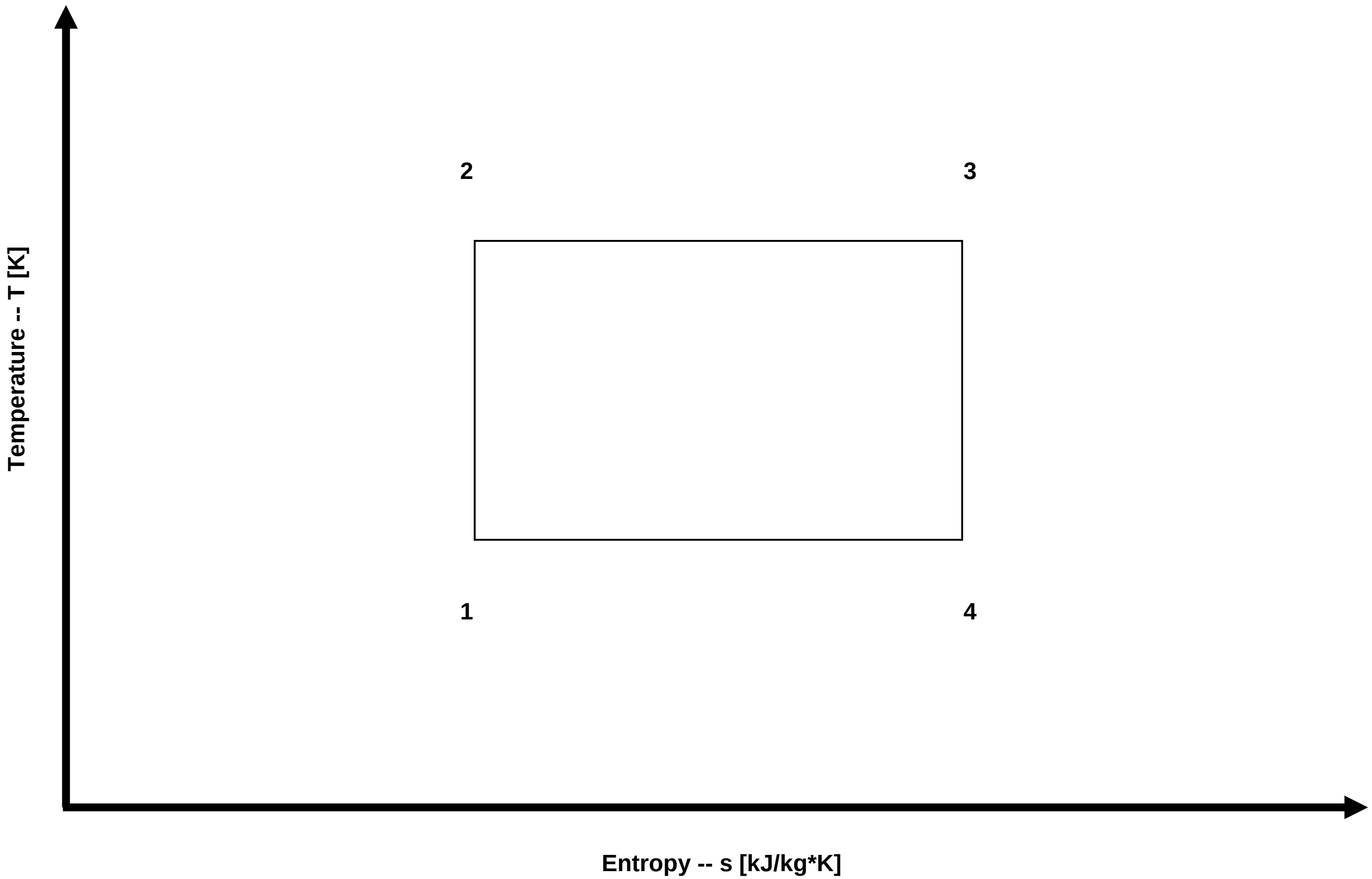
For air: χ = 1.4 [/], R = 0.2867 [kJ/kg*K] and c_p = 1.004 [kJ/kg*K]

Carnot Cycle



Carnot Cycle Schematic Layout

Carnot Cycle



Carnot Cycle T - s Diagram

Carnot Cycle

The thermal cycle efficiency can be given as a function of specific external work (specific net power output) and heat added to the working fluid as follows:

$\eta = w/q_h = (w_t - w_c)/q_h = (q_h - q_l)/q_h = 1 - q_l/q_h = 1 - T_1\Delta s/T_2\Delta s = 1 - T_1/T_2 = 1 - T_R/T_A$

where

η - thermal efficiency [/]

w - specific external work (specific net power output) [kJ/kg]

w_t - expansion specific power output [kJ/kg]

w_c - compression specific power input [kJ/kg]

q_h - heat added to the working fluid [kJ/kg]

Carnot Cycle

q_l - heat rejected from the working fluid [kJ/kg]

Δs - entropy change during heat addition and heat rejection [kJ/kg*K]

T_A - temperature during heat addition [K]

T_R - temperature during heat rejection [K]

For isentropic compression and expansion:

$T_2/T_1 = (p_2/p_1)^{(\varkappa-1)/\varkappa} = T_3/T_4 = (p_3/p_4)^{(\varkappa-1)/\varkappa}$

and

$\varkappa = c_p/c_v$ - for air $\varkappa = 1.4$ [/]

p_1, p_2, p_3, p_4 - pressure values at points 1, 2, 3 and 4 [atm]

T_1, T_2, T_3, T_4 - temperature values at points 1, 2, 3 and 4 [K]

Carnot Cycle

Again, it follows that

$$\eta = 1 - T_1/T_2 = 1 - T_R/T_A$$

The Carnot Cycle efficiency is not dependent on the working fluid properties.

Carnot Cycle

Governing Equations

$T_2/T_1 = (p_2/p_1)^{(\varkappa-1)/\varkappa}$

$T_3/T_4 = (p_3/p_4)^{(\varkappa-1)/\varkappa}$

$\varkappa = c_p/c_v$

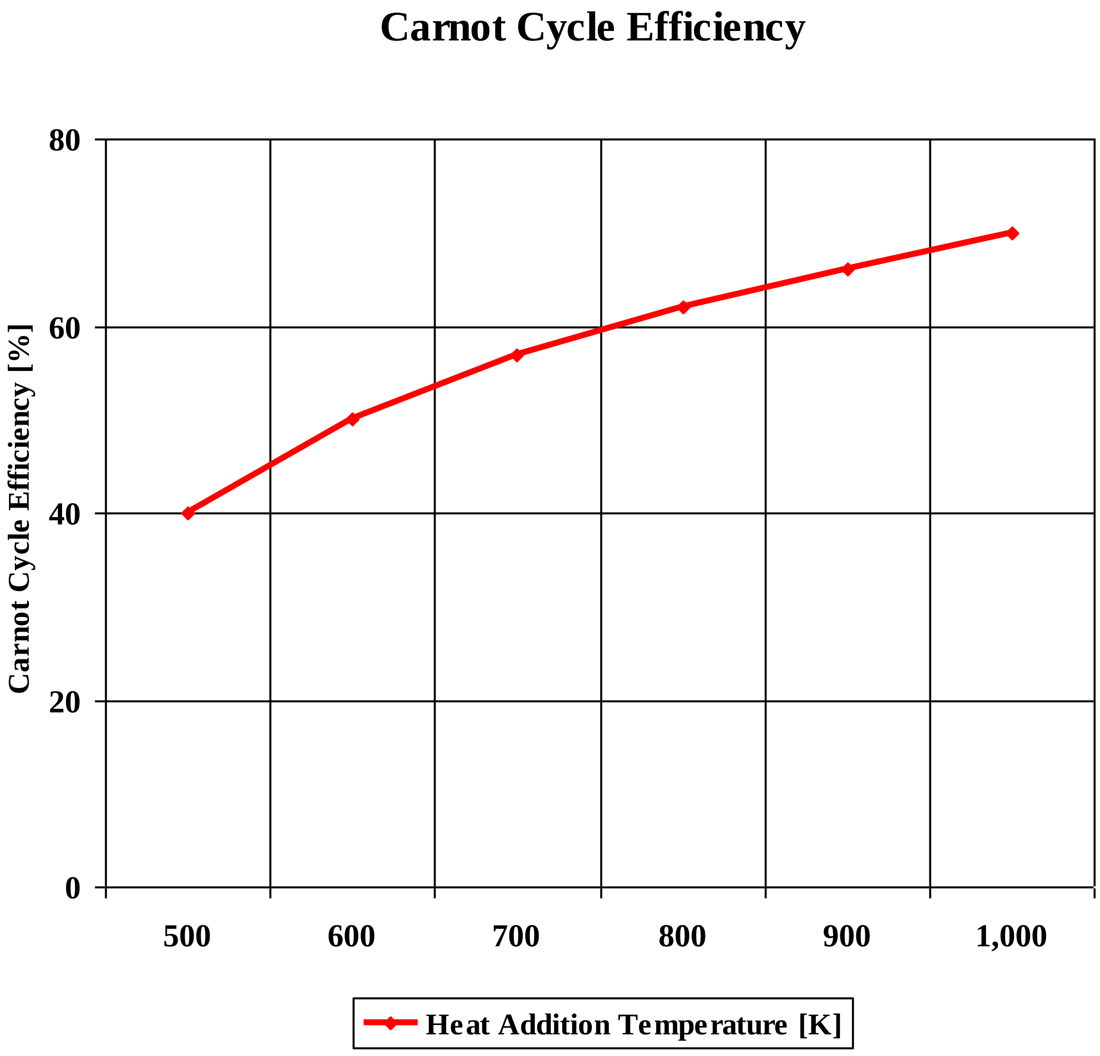
$pv = RT$

$q_h = T_2 \Delta s = T_A \Delta s$

$q_l = T_1 \Delta s = T_R \Delta s$

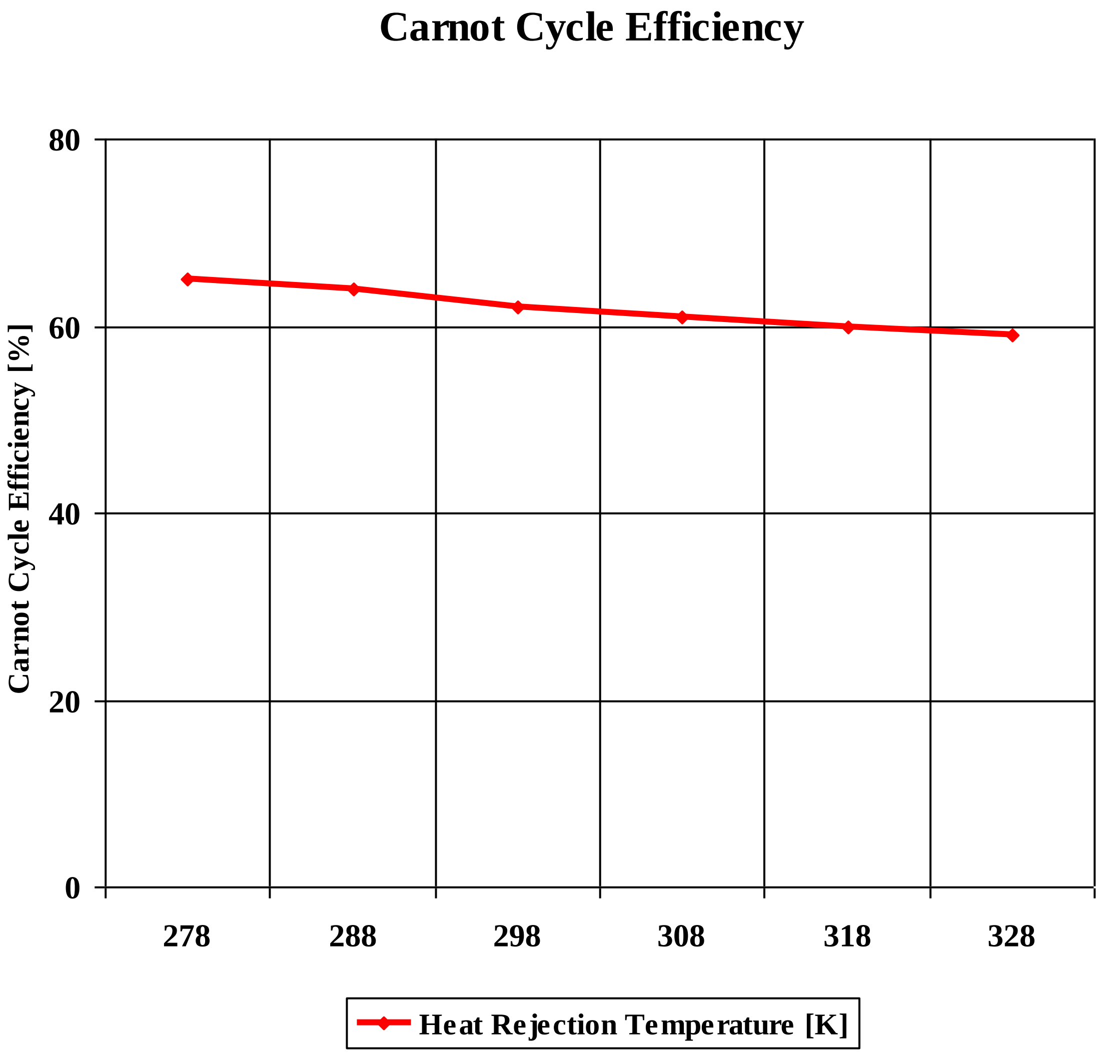
$\eta = 1 - T_1/T_2 = 1 - T_R/T_A$

Carnot Cycle



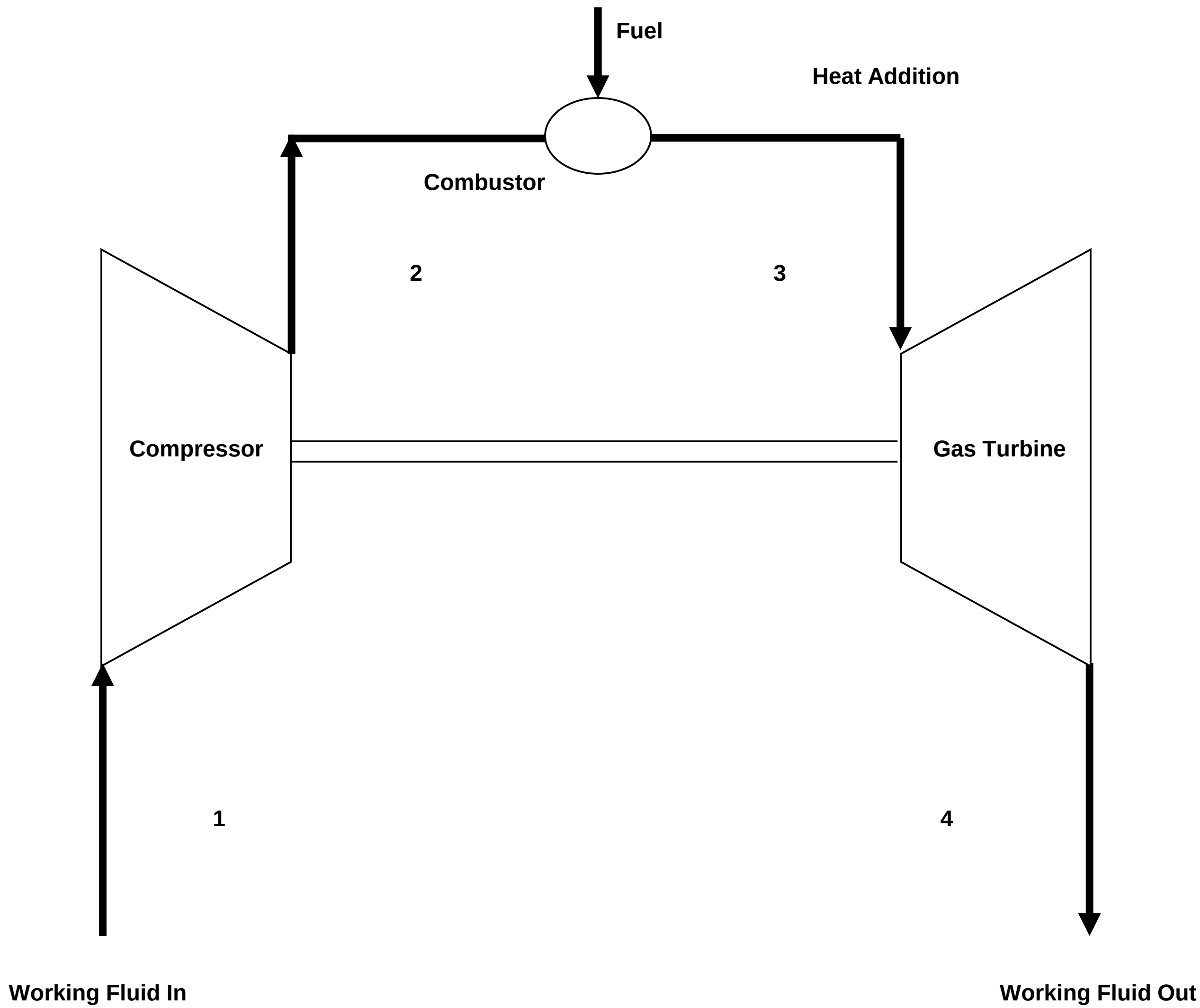
Compressor Inlet Temperature: 298 [K]

Carnot Cycle



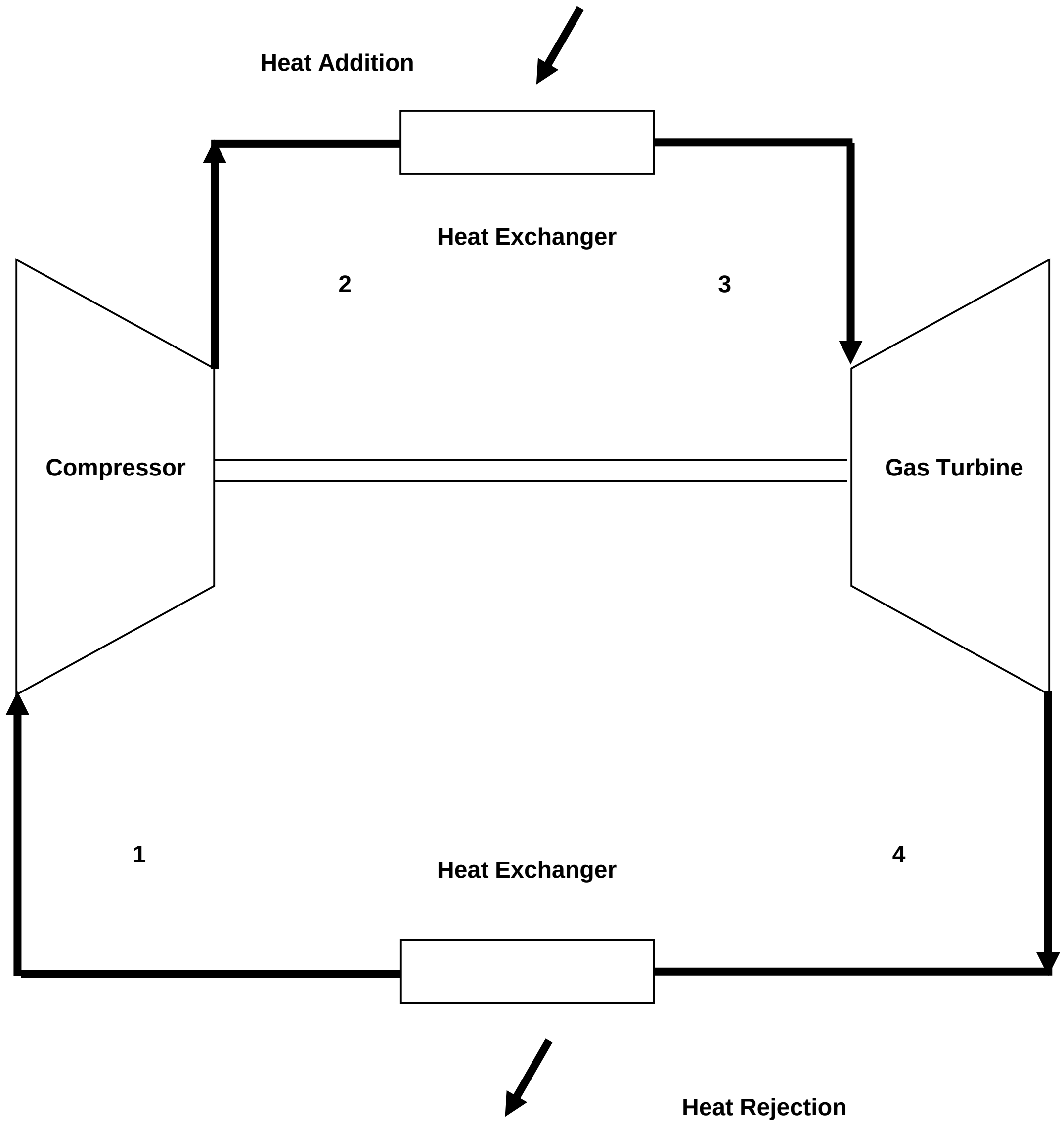
Turbine Inlet Temperature: 800 [K]

Brayton Cycle (Gas Turbine)



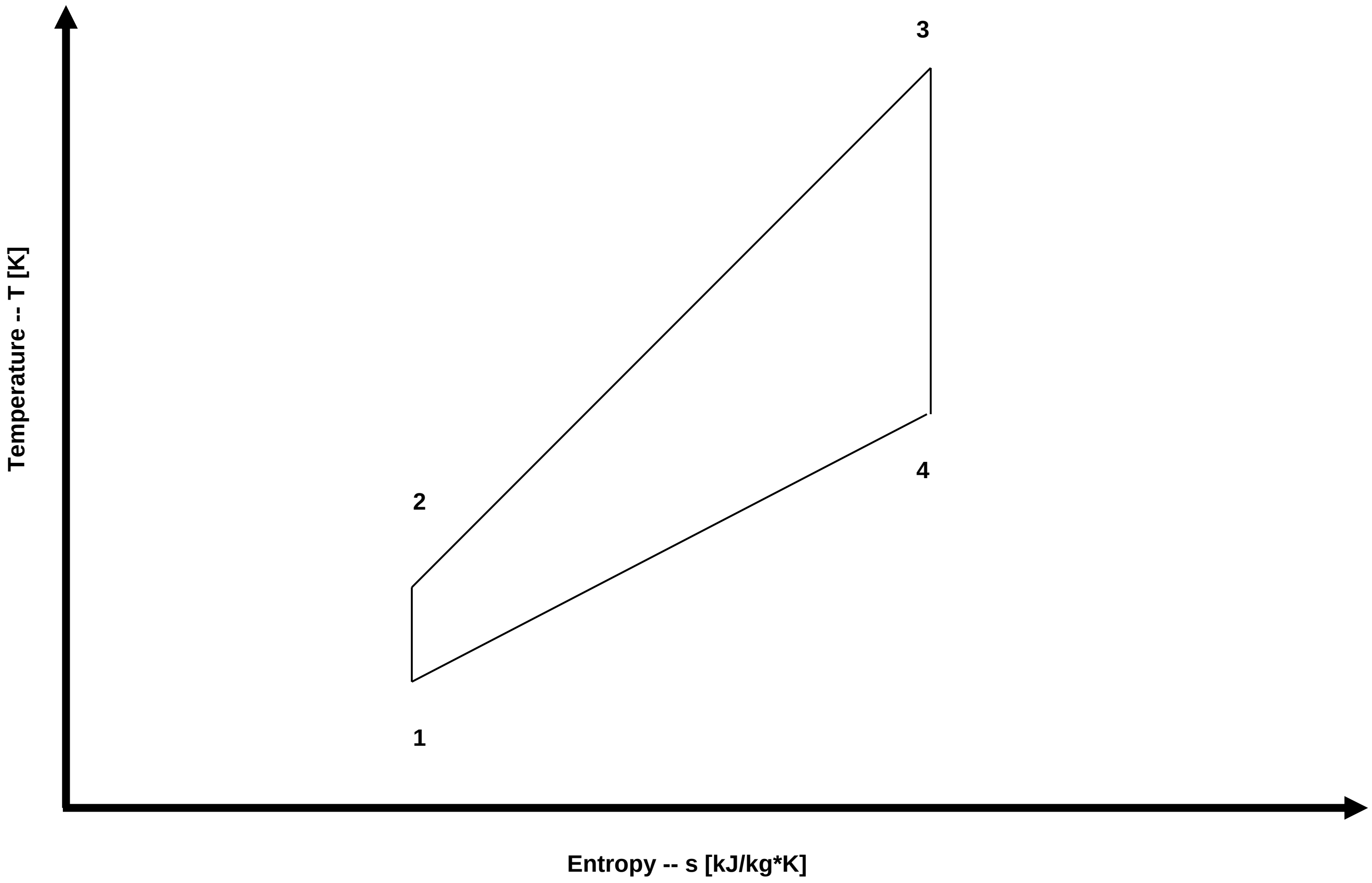
Brayton Cycle (Gas Turbine) Schematic Layout -- Open Cycle

Brayton Cycle (Gas Turbine)



Brayton Cycle Schematic Layout -- Closed Cycle

Brayton Cycle (Gas Turbine)



Brayton Cycle (Gas Turbine) T - s Diagram

Brayton Cycle (Gas Turbine)

The thermal cycle efficiency can be given as a function of specific external work (specific net power output) and heat added to the working fluid as follows:

$\eta = w/q_h = (w_t - w_c)/q_h = (q_h - q_l)/q_h$

or

$\eta = 1 - q_l/q_h = 1 - (c_p(T_4 - T_1))/(c_p(T_3 - T_2)) = 1 - (T_1(T_4/T_1 - 1))/(T_2(T_3/T_2 - 1))$

where

η - thermal efficiency [/]

w - specific external work (specific net power output) [kJ/kg]

w_t - expansion specific power output [kJ/kg]

w_c - compression specific power input [kJ/kg]

W - external work (net power output) [kW]

Brayton Cycle (Gas Turbine)

W_t - expansion power output [kW]

W_c - compression power input [kW]

q_h - heat added to the working fluid [kJ/kg]

q_l - heat rejected from the working fluid [kJ/kg]

c_p - specific heat at constant pressure [kJ/kg*K]

c_v - specific heat at constant volume [kJ/kg*K]

m - working fluid mass flow rate [kg/s]

r_p - compression ratio [/]

For isentropic compression and expansion:

$T_2/T_1 = (p_2/p_1)^{(\varkappa-1)/\varkappa}$

$T_3/T_4 = (p_3/p_4)^{(\varkappa-1)/\varkappa}$

Brayton Cycle (Gas Turbine)

Knowing that

$p_3/p_4 = p_2/p_1$

where

$\varkappa = c_p/c_v$ - for air $\varkappa = 1.4$ [/]

p_1, p_2, p_3, p_4 - pressure values at points 1, 2, 3 and 4 [atm]

T_1, T_2, T_3, T_4 - temperature values at points 1, 2, 3 and 4 [K]

It follows that

$T_3/T_4 = T_2/T_1$

or

$T_3/T_2 = T_4/T_1$

Brayton Cycle (Gas Turbine)

Therefore, after some mathematical operations the thermal efficiency is:

$\eta = 1 - T_1/T_2 = 1 - T_4/T_3$

If the temperature ratio is substituted in terms of the compression ratio:

$\eta = 1 - 1/r_p^{(\varkappa-1)/\varkappa}$

where

$r_p = p_2/p_1$

Brayton Cycle (Gas Turbine)

Governing Equations

$T_2/T_1 = (p_2/p_1)^{(\varkappa-1/)\varkappa}$

$p_2/p_1 = (T_2/T_1)^{\varkappa/(\varkappa-1)}$

$T_3/T_4 = (p_3/p_4)^{(\varkappa-1/)\varkappa}$

$p_3/p_4 = (T_3/T_4)^{\varkappa/(\varkappa-1)}$

$\varkappa = c_p/c_v$

$pv = RT$

$w = q_h - q_l$

$q_h = c_p(T_3 - T_2)$

$q_l = c_p(T_4 - T_1)$

Brayton Cycle (Gas Turbine)

Governing Equations (Continued)

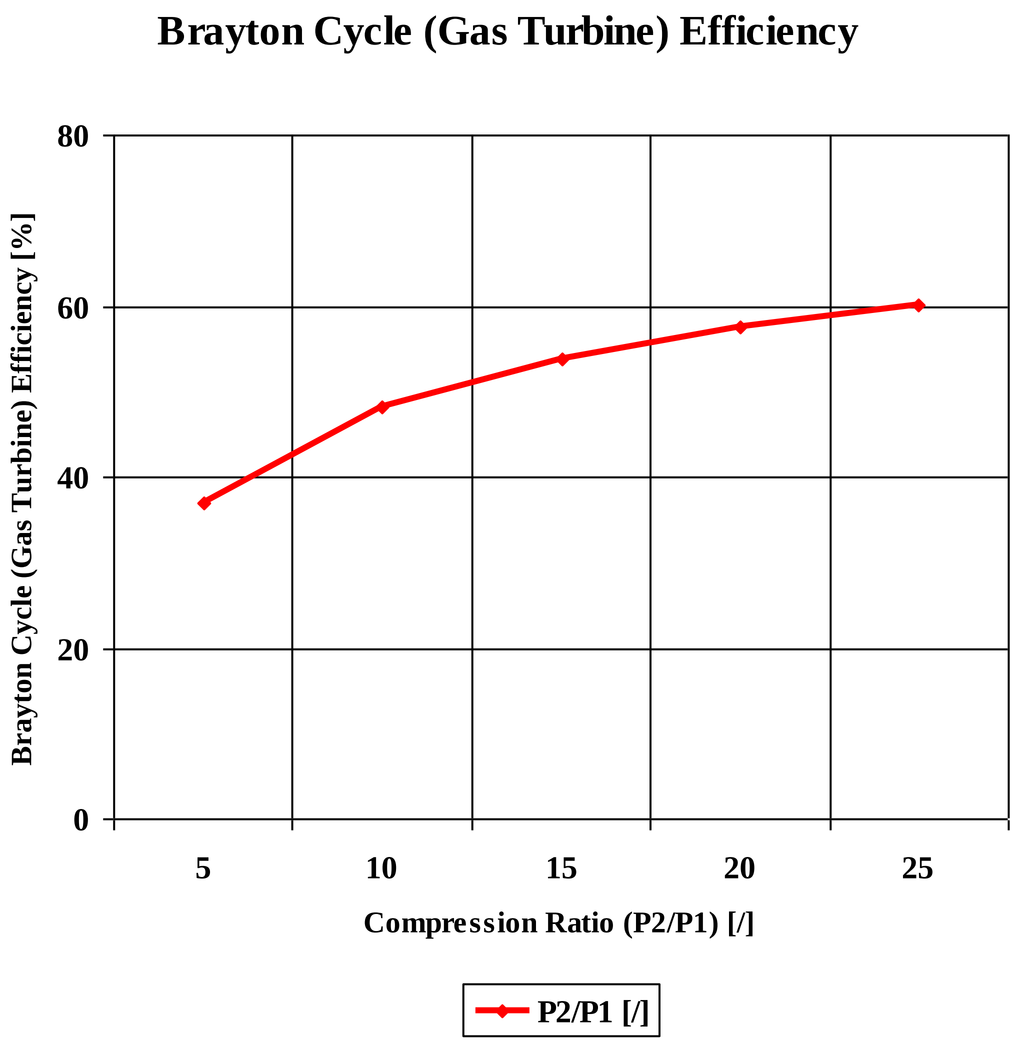
$w = c_p(T_3 - T_2) - c_p(T_4 - T_1)$

$W = (c_p(T_3 - T_2) - c_p(T_4 - T_1))m$

$\eta = 1 - 1/r_p^{(\varkappa-1)/\varkappa}$

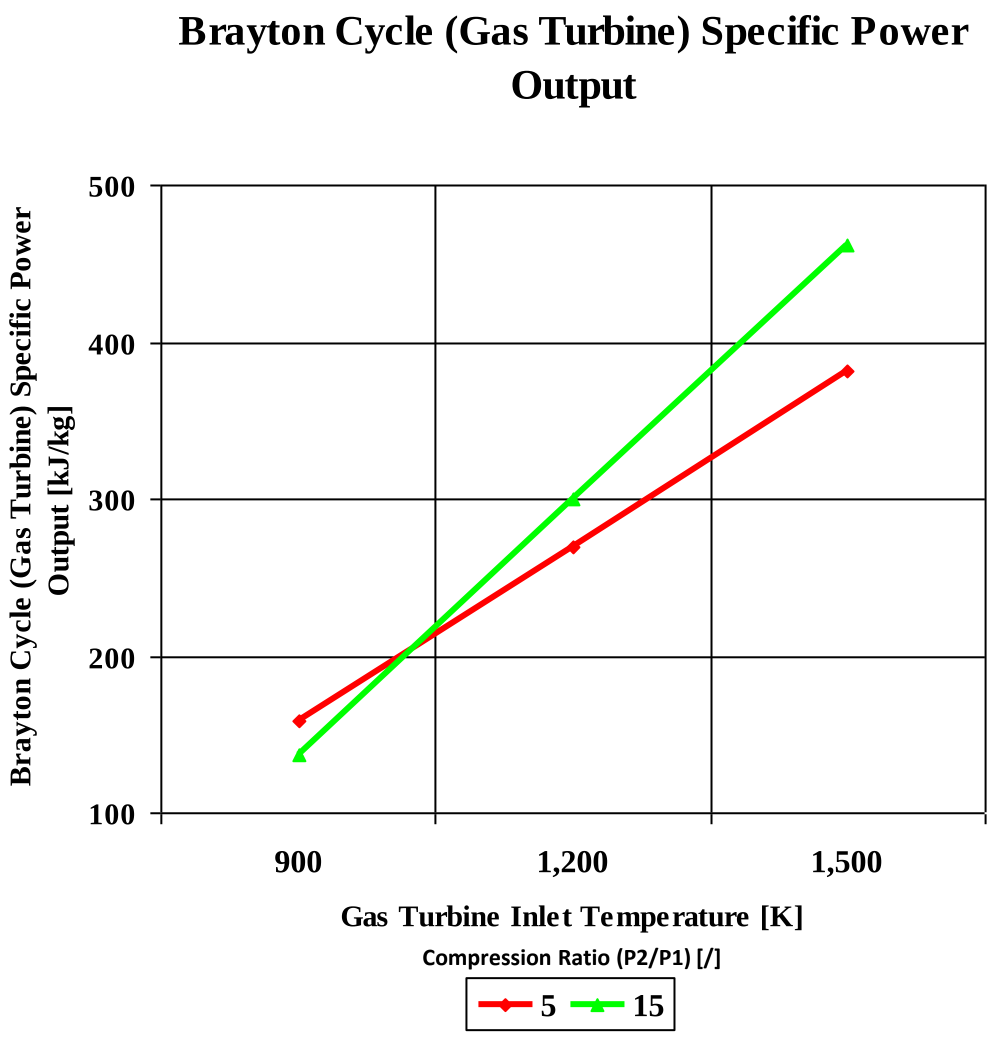
$r_p = p_2/p_1$

Brayton Cycle (Gas Turbine)



Working Fluid: Air

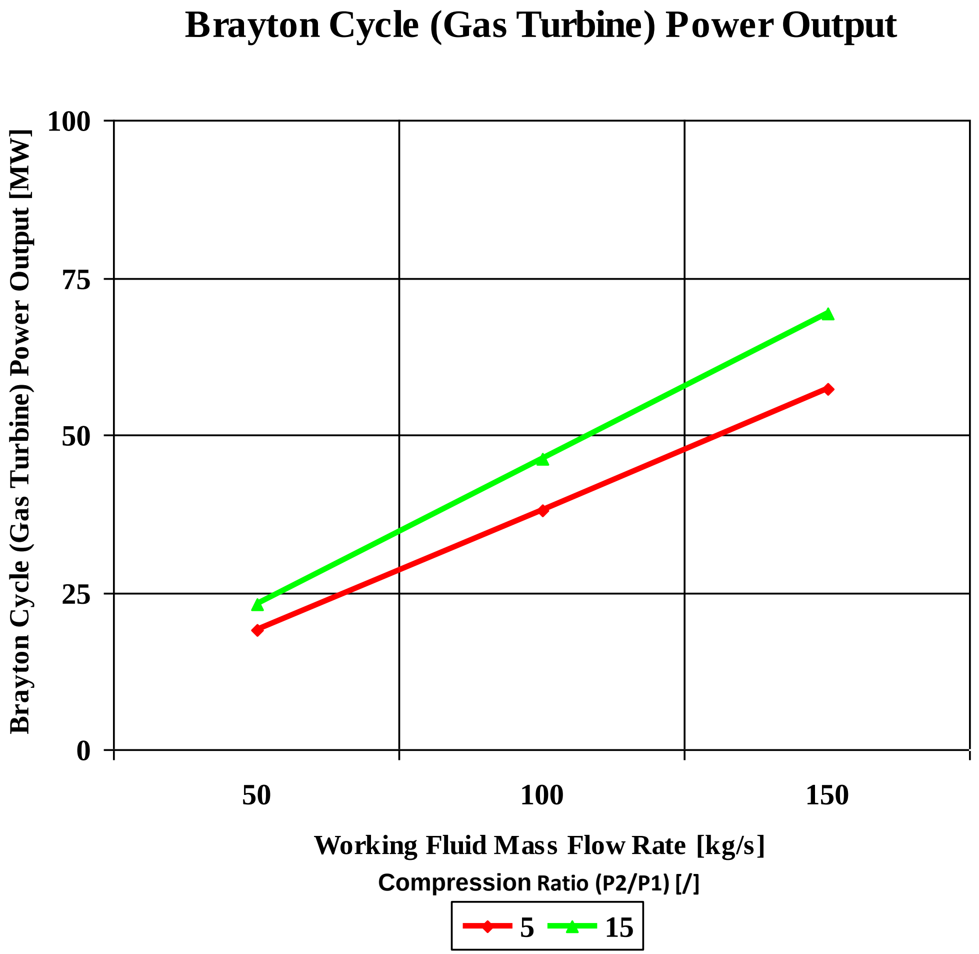
Brayton Cycle (Gas Turbine)



Working Fluid: Air

Compressor Inlet Temperature: 298 [K]

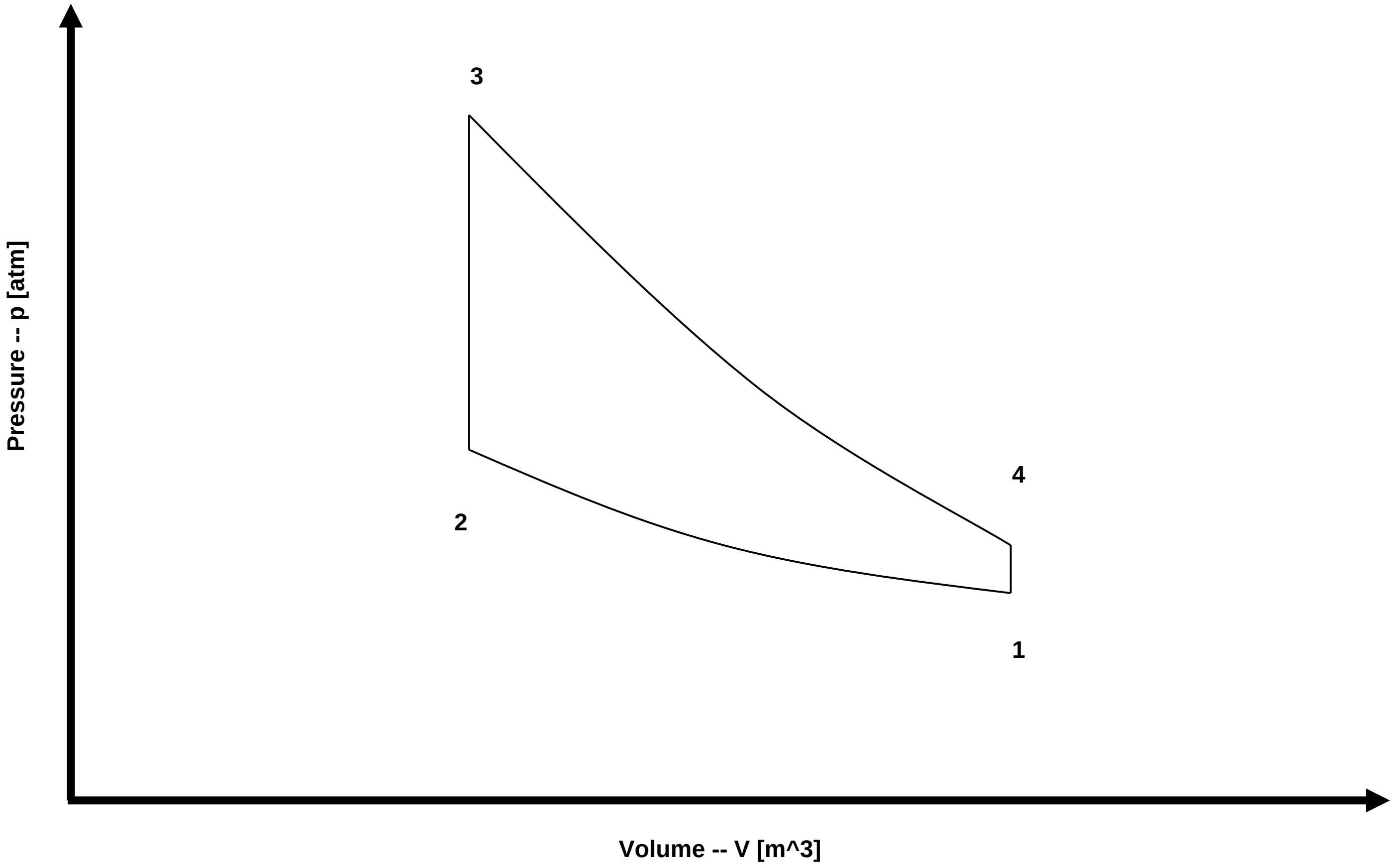
Brayton Cycle (Gas Turbine)



Working Fluid: Air

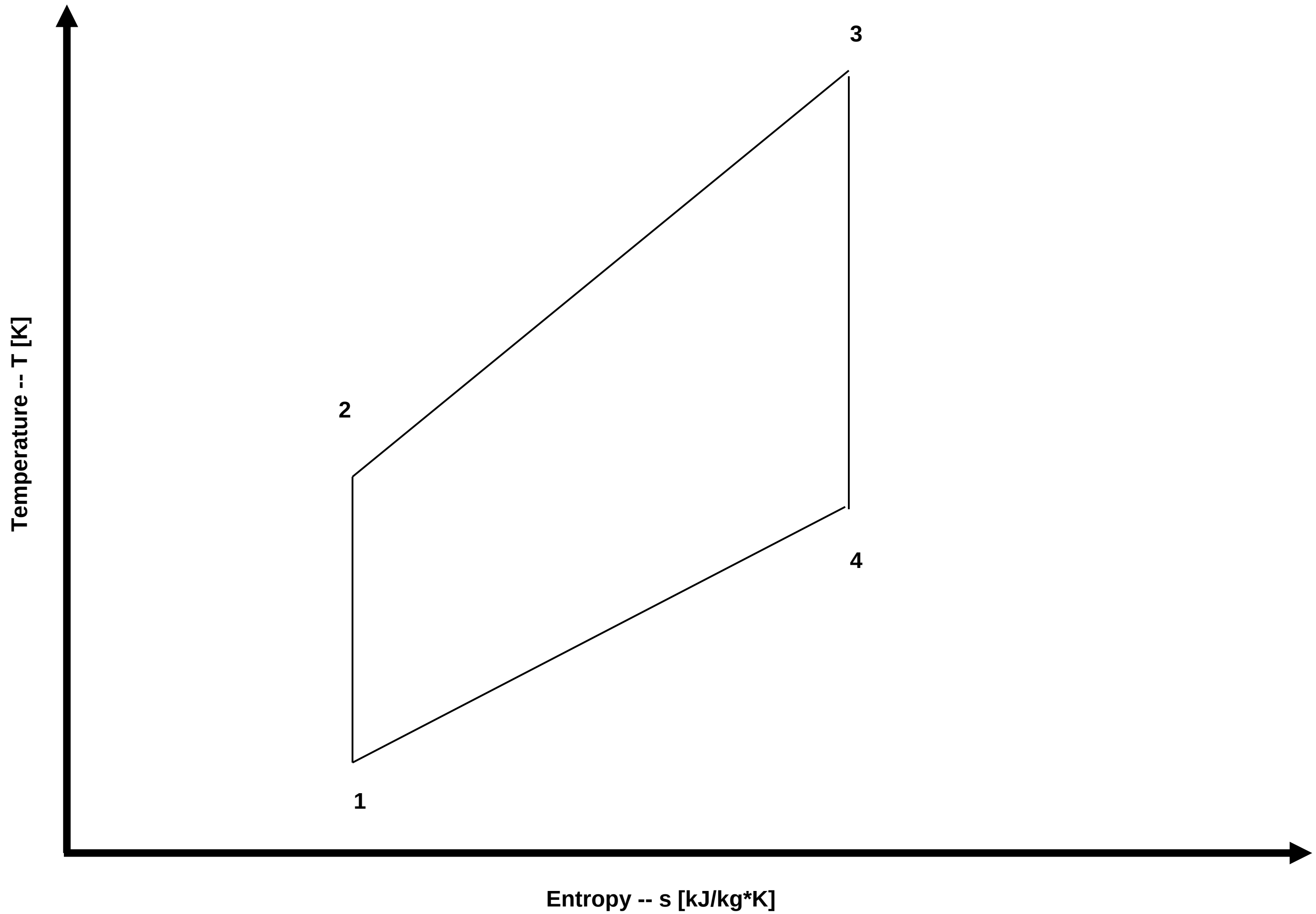
Compressor Inlet Temperature: 298 [K] -- Gas Turbine Inlet Temperature: 1,500 [K]

Otto Cycle



Otto Cycle p - V Diagram

Otto Cycle



Otto Cycle T - s Diagram

Otto Cycle

The thermal cycle efficiency can be given as a function of specific external work (specific net power output) and heat added to the working fluid as follows:

$\eta = w/q_h = (w_e - w_c)/q_h = (q_h - q_l)/q_h$

or

$\eta = 1 - q_l/q_h = 1 - (c_v(T_4 - T_1))/(c_v(T_3 - T_2)) = 1 - (T_1(T_4/T_1 - 1))/(T_2(T_3/T_2 - 1))$

where

η - thermal efficiency [/]

w - specific external work (specific net power output) [kJ/kg]

w_e - expansion specific power output [kJ/kg]

Otto Cycle

w_c - compression specific power input [kJ/kg]

W - external work (net power output) [kW]

W_e - expansion power output [kW]

W_c - compression power input [kW]

q_h - heat added to the working fluid [kJ/kg]

q_l - heat rejected from the working fluid [kJ/kg]

c_p - specific heat at constant pressure [kJ/kg*K]

c_v - specific heat at constant volume [kJ/kg*K]

m - working fluid mass flow rate [kg/s]

ε - compression ratio [/]

Otto Cycle

For isentropic compression and expansion:

$T_2/T_1 = (p_2/p_1)^{(\varkappa-1)/\varkappa} = (V_1/V_2)^{(\varkappa-1)}$

$T_4/T_3 = (p_4/p_3)^{(\varkappa-1)/\varkappa} = (V_3/V_4)^{(\varkappa-1)}$

Knowing that

$V_3/V_4 = V_2/V_1$

where

$\varkappa = c_p/c_v$ - for air $\varkappa = 1.4$ [/]

V_1, V_2, V_3, V_4 - volume values at points 1, 2, 3 and 4 [m^3]

p_1, p_2, p_3, p_4 - pressure values at points 1, 2, 3 and 4 [atm]

T_1, T_2, T_3, T_4 - temperature values at points 1, 2, 3 and 4 [K]

Otto Cycle

It follows that

$T_3/T_4 = T_2/T_1 = (V_1/V_2)^{(\varkappa-1)} = \varepsilon^{(\varkappa-1)}$

where

$\varepsilon = V_1/V_2$

Therefore, after some mathematical operations the thermal efficiency is:

$\eta = 1 - T_1/T_2 = 1 - 1/(V_1/V_2)^{(\varkappa-1)}$

If the temperature ratio is substituted in terms of the volume/compression ratio:

$\eta = 1 - 1/\varepsilon^{(\varkappa-1)}$

Otto Cycle

Governing Equations

$T_2/T_1 = (V_1/V_2)^{(\varkappa-1)}$

$V_1/V_2 = (T_2/T_1)^{1/(\varkappa-1)}$

$T_3/T_4 = (V_4/V_3)^{(\varkappa-1)}$

$V_4/V_3 = (T_3/T_4)^{1/(\varkappa-1)}$

$\varkappa = c_p/c_v$

$pv = RT$

$w = q_h - q_l$

$q_h = c_v(T_3 - T_2)$

$q_l = c_v(T_4 - T_1)$

Otto Cycle

Governing Equations (Continued)

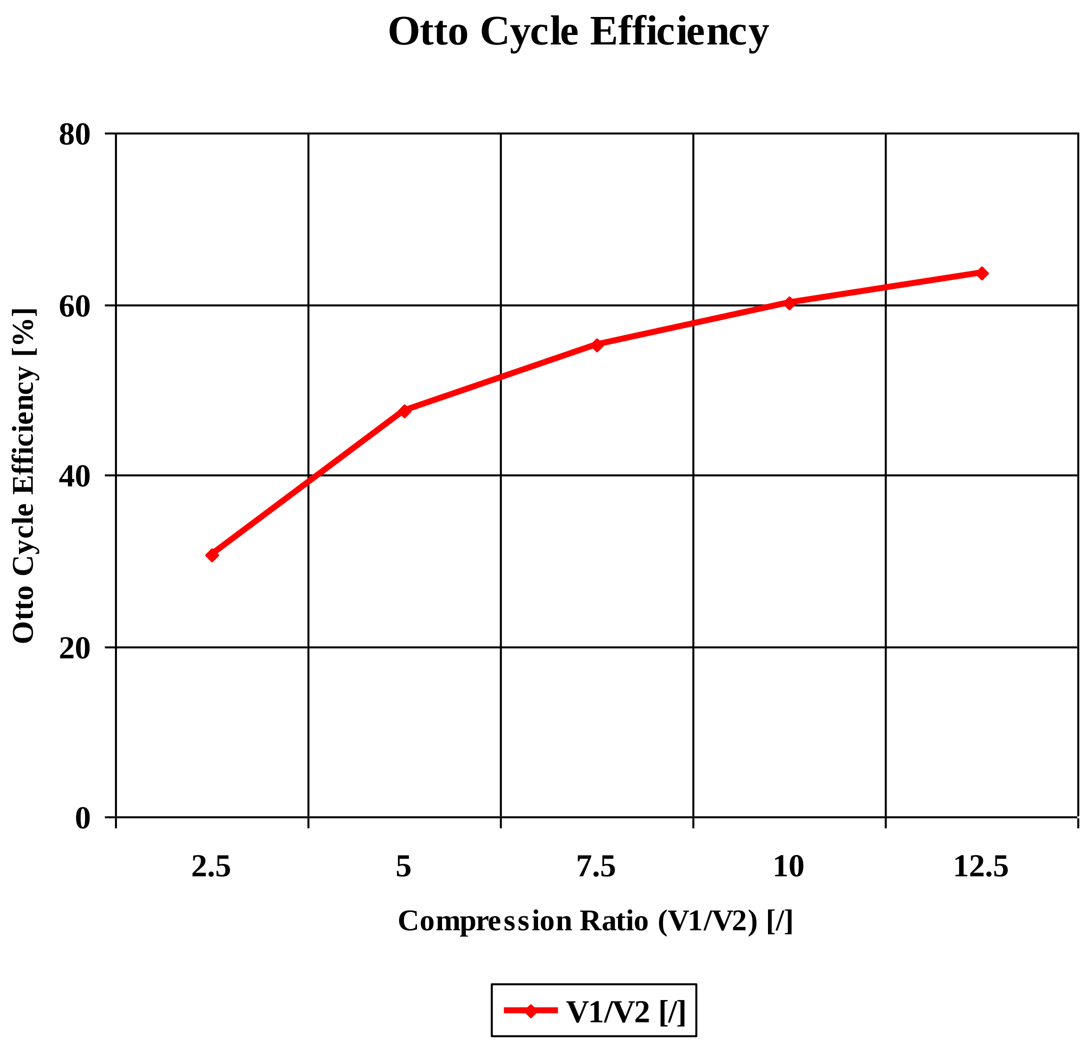
$w = c_v(T_3 - T_2) - c_v(T_4 - T_1)$

$W = (c_v(T_3 - T_2) - c_v(T_4 - T_1))m$

$\eta = 1 - 1/\varepsilon^{(\varkappa - 1)}$

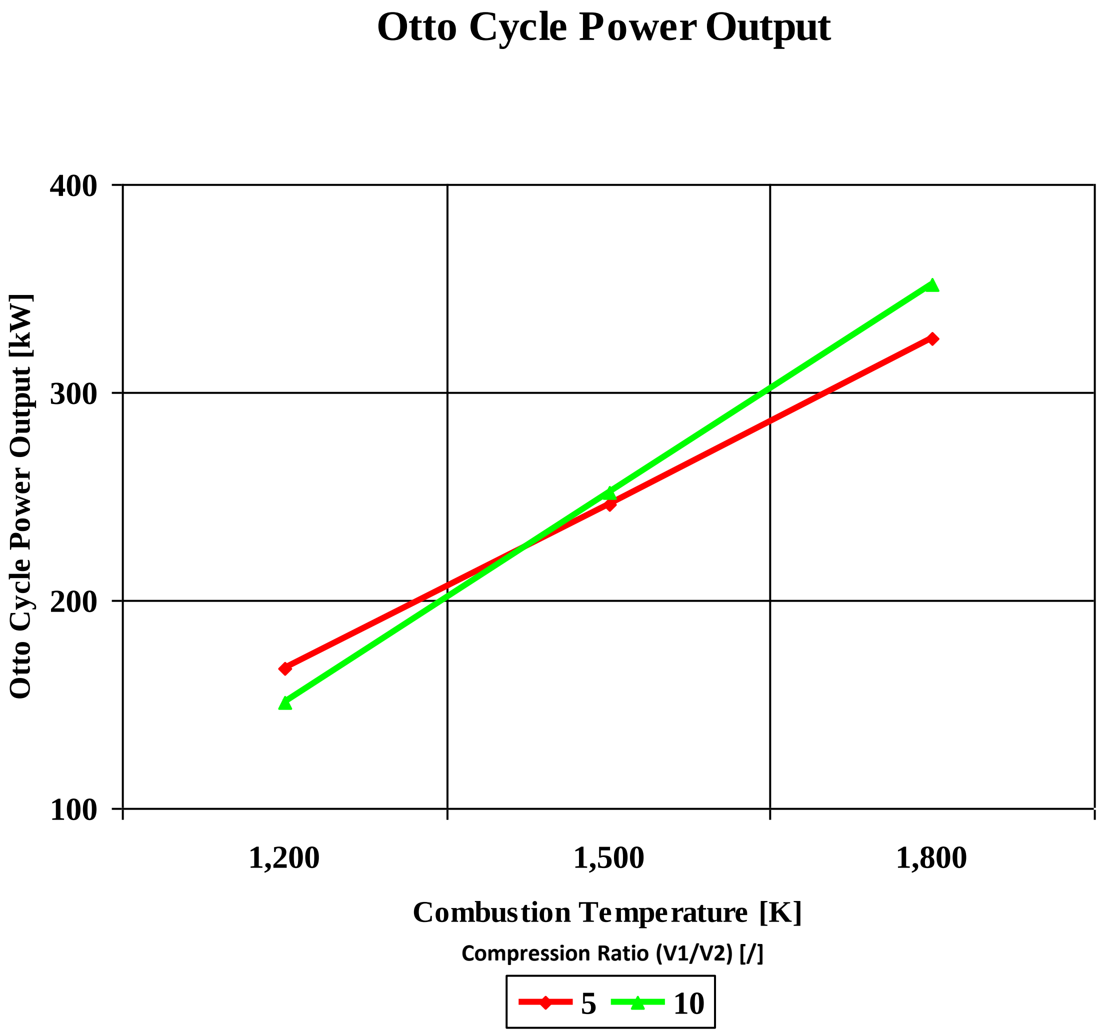
$\varepsilon = V_1/V_2$

Otto Cycle



Working Fluid: Air

Otto Cycle

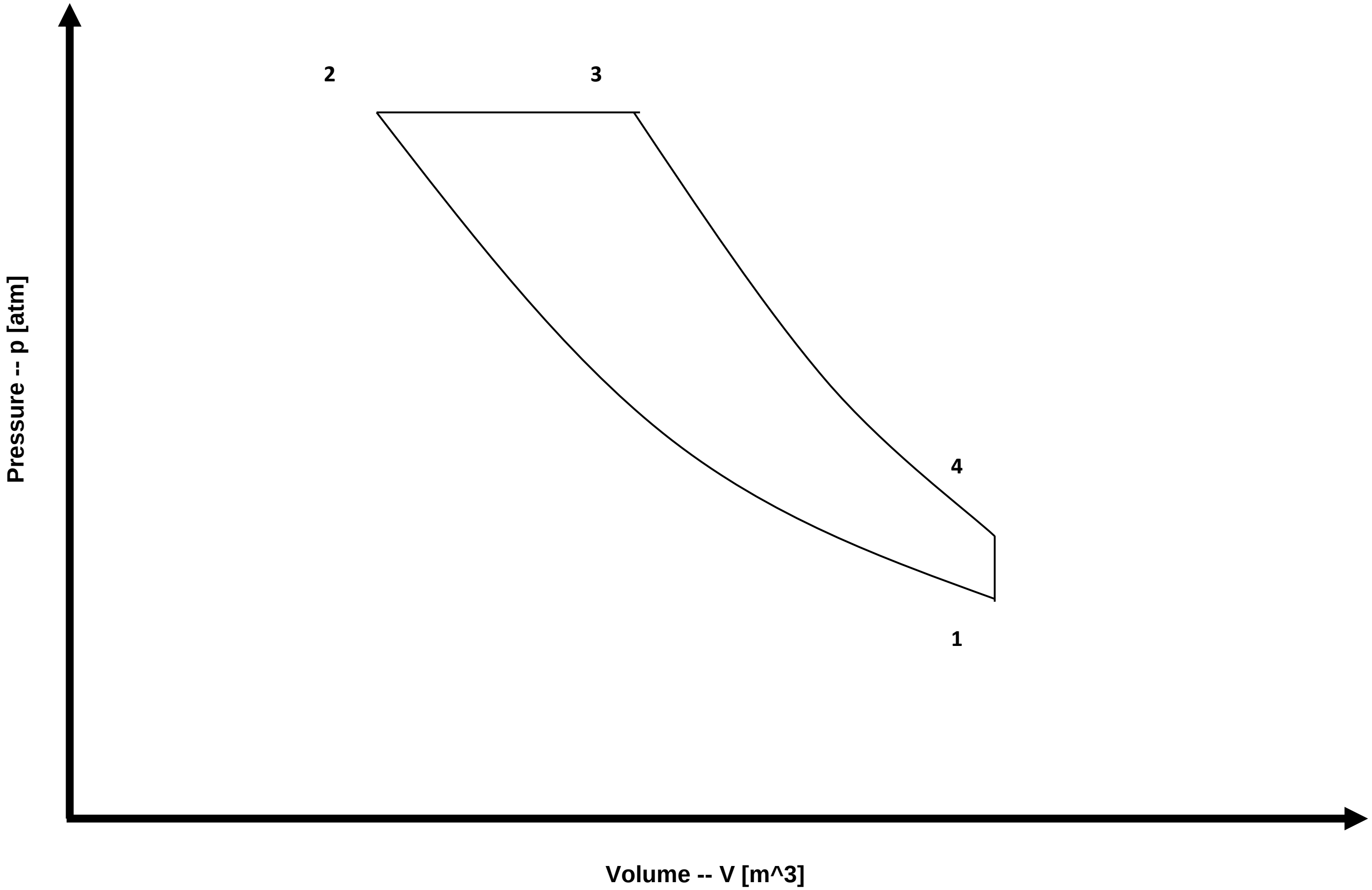


Working Fluid: Air

Ambient Temperature: 298 [K] -- Number of Revolutions: 60 [1/s]

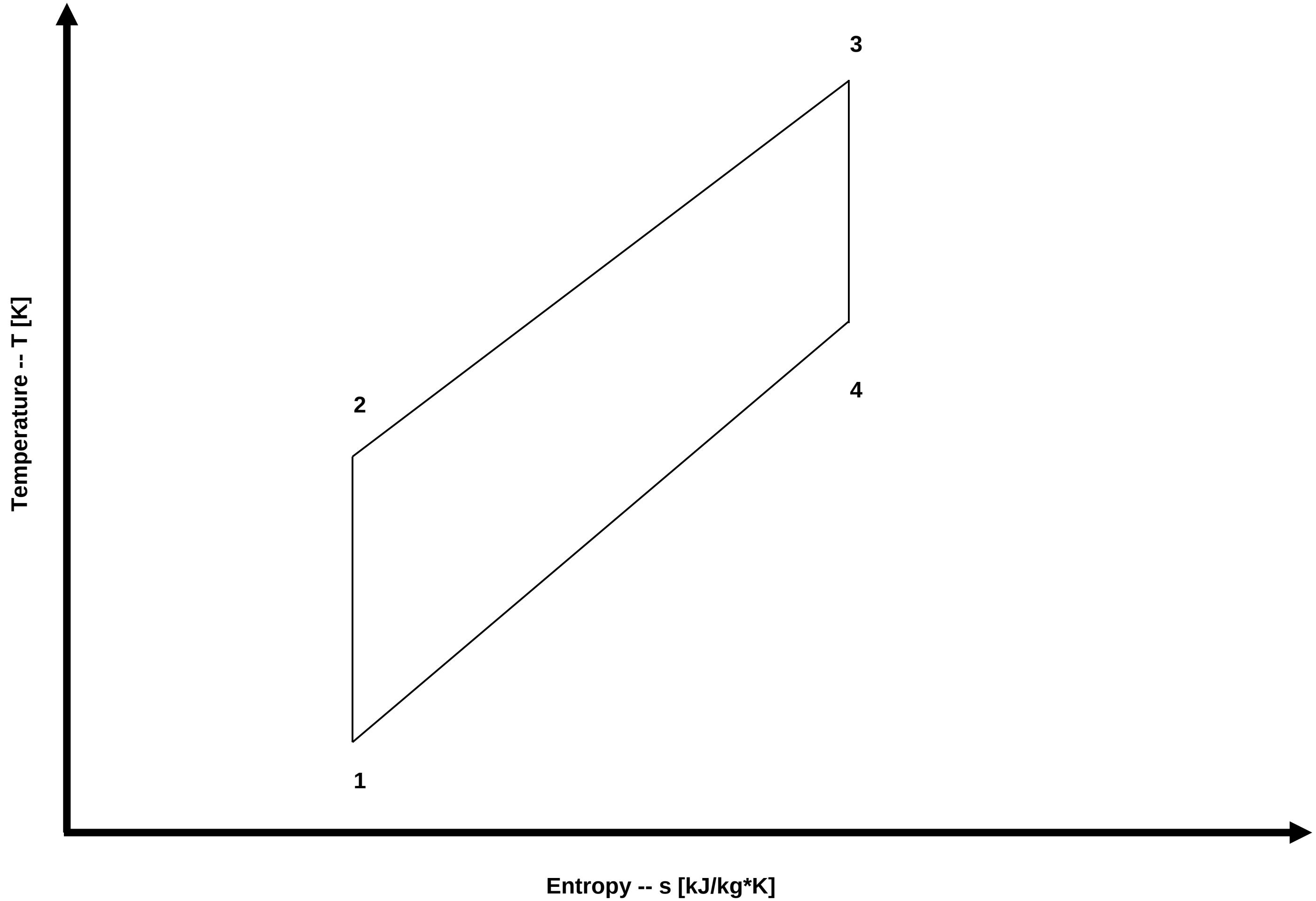
For a Given Geometry of a Four Cylinder and Four Stroke Otto Engine

Diesel Cycle



Diesel Cycle p - V Diagram

Diesel Cycle



Diesel Cycle T - s Diagram

Diesel Cycle

The thermal cycle efficiency can be given as a function of specific external work (specific net power output) and heat added to the working fluid as follows:

$\eta = w/q_h = (w_e - w_c)/q_h = (q_h - q_l)/q_h$

or

$\eta = 1 - q_l/q_h = 1 - (c_v(T_4 - T_1))/(c_p(T_3 - T_2)) = 1 - (T_1(T_4/T_1 - 1))/(\varkappa T_2(T_3/T_2 - 1))$

where

η - thermal efficiency [/]

w - specific external work (specific net power output) [kJ/kg]

w_e - expansion specific power output [kJ/kg]

Diesel Cycle

w_c - compression specific power input [kJ/kg]

W - external work (net power output) [kW]

W_e - expansion power output [kW]

W_c - compression power input [kW]

q_h - heat added to the working fluid [kJ/kg]

q_l - heat rejected from the working fluid [kJ/kg]

c_p - specific heat at constant pressure [kJ/kg*K]

c_v - specific heat at constant volume [kJ/kg*K]

m - working fluid mass flow rate [kg/s]

ε - compression ratio [/]

Diesel Cycle

φ - cut off ratio [/]

For isentropic compression and expansion:

$T_2/T_1 = (p_2/p_1)^{(\varkappa-1)/\varkappa} = (V_1/V_2)^{(\varkappa-1)}$

$T_4/T_3 = (p_4/p_3)^{(\varkappa-1)/\varkappa} = (V_3/V_4)^{(\varkappa-1)}$

where

$\varkappa = c_p/c_v$ - for air $\varkappa = 1.4$ [/]

V_1, V_2, V_3, V_4 - volume values at points 1, 2, 3 and 4 [m^3]

p_1, p_2, p_3, p_4 - pressure values at points 1, 2, 3 and 4 [atm]

T_1, T_2, T_3, T_4 - temperature values at points 1, 2, 3 and 4 [K]

Diesel Cycle

Knowing that

$s_3 - s_2 = s_4 - s_1$

and

$s_3 - s_2 = c_p \ln(T_3/T_2)$

$s_4 - s_1 = c_v \ln(T_4/T_1)$

s_1, s_2, s_3, s_4 - specific entropy values at points 1, 2, 3 and 4 [kJ/kg*K]

It follows

$(T_3/T_2)^{\varkappa} = T_4/T_1$

It follows that

$T_3/T_4 = T_2/T_1 = (V_1/V_2)^{(\varkappa-1)} = \varepsilon^{(\varkappa-1)}$

Diesel Cycle

When combustion takes place at a constant pressure:

$T_3/T_2 = V_3/V_2$

where

$\varepsilon = V_1/V_2$

$\varphi = V_3/V_2$

Therefore, after some mathematical operations the thermal efficiency is:

$\eta = 1 - (T_1((T_3/T_2)^{\varkappa} - 1)))/(\varkappa T_2(T_3/T_2 - 1))$

If the temperature ratio is substituted in terms of the volume/compression ratio:

$\eta = 1 - (\varphi^{\varkappa} - 1)/(\varkappa \varepsilon^{(\varkappa-1)}(\varphi - 1))$

Diesel Cycle

Governing Equations

$T_2/T_1 = (V_1/V_2)^{(\varkappa-1)}$

$V_1/V_2 = (T_2/T_1)^{1/(\varkappa-1)}$

$T_3/T_4 = (V_4/V_3)^{(\varkappa-1)}$

$V_4/V_3 = (T_3/T_4)^{1/(\varkappa-1)}$

$\varkappa = c_p/c_v$

$pv = RT$

$w = q_h - q_l$

$q_h = c_p(T_3 - T_2)$

$q_l = c_v(T_4 - T_1)$

Diesel Cycle

Governing Equations (Continued)

$w = c_p(T_3 - T_2) - c_v(T_4 - T_1)$

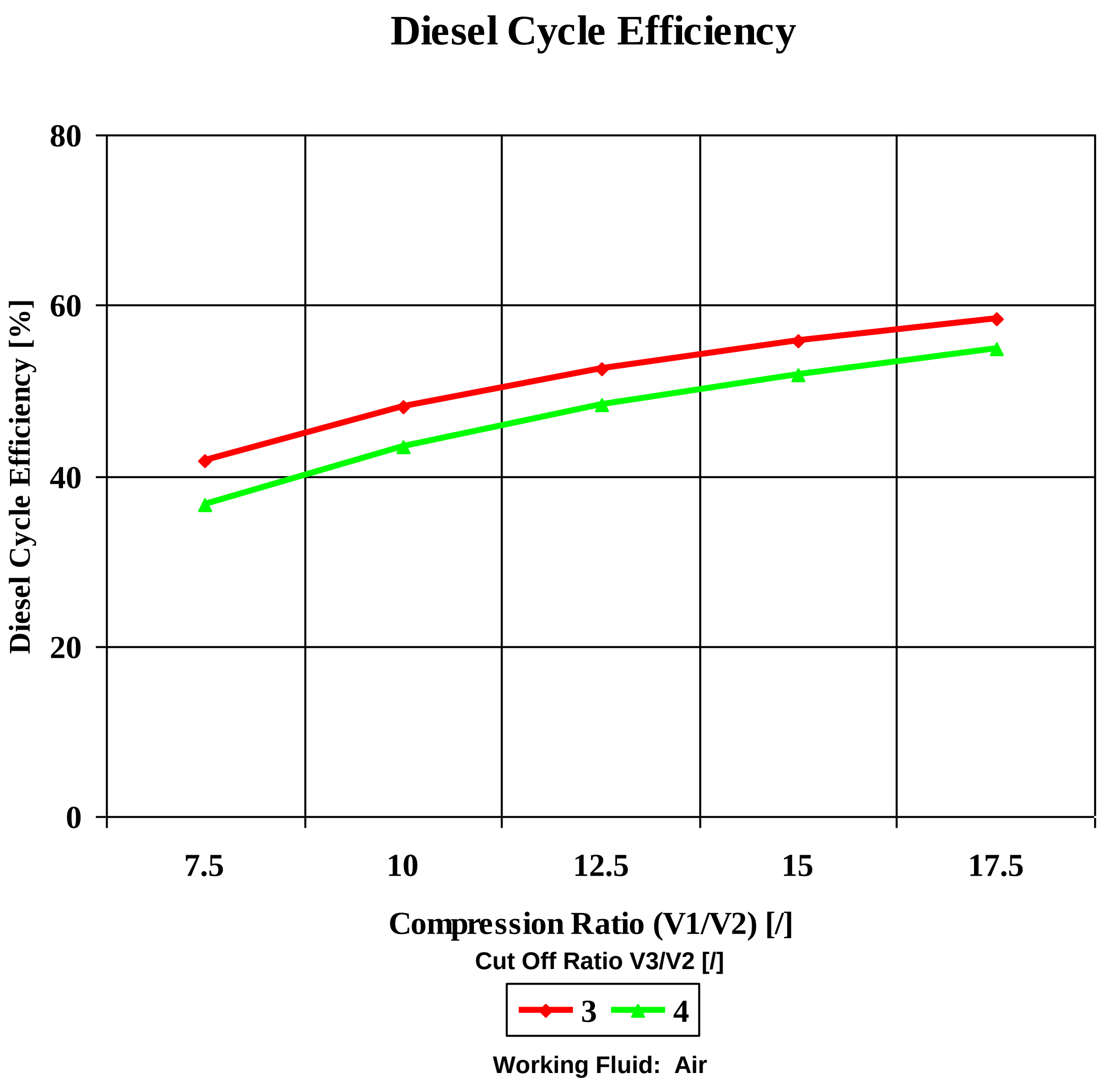
$W = (c_p(T_3 - T_2) - c_v(T_4 - T_1))m$

$\eta = 1 - (\varphi^{\varkappa} - 1)/(\varkappa\varepsilon^{(\varkappa-1)}(\varphi - 1))$

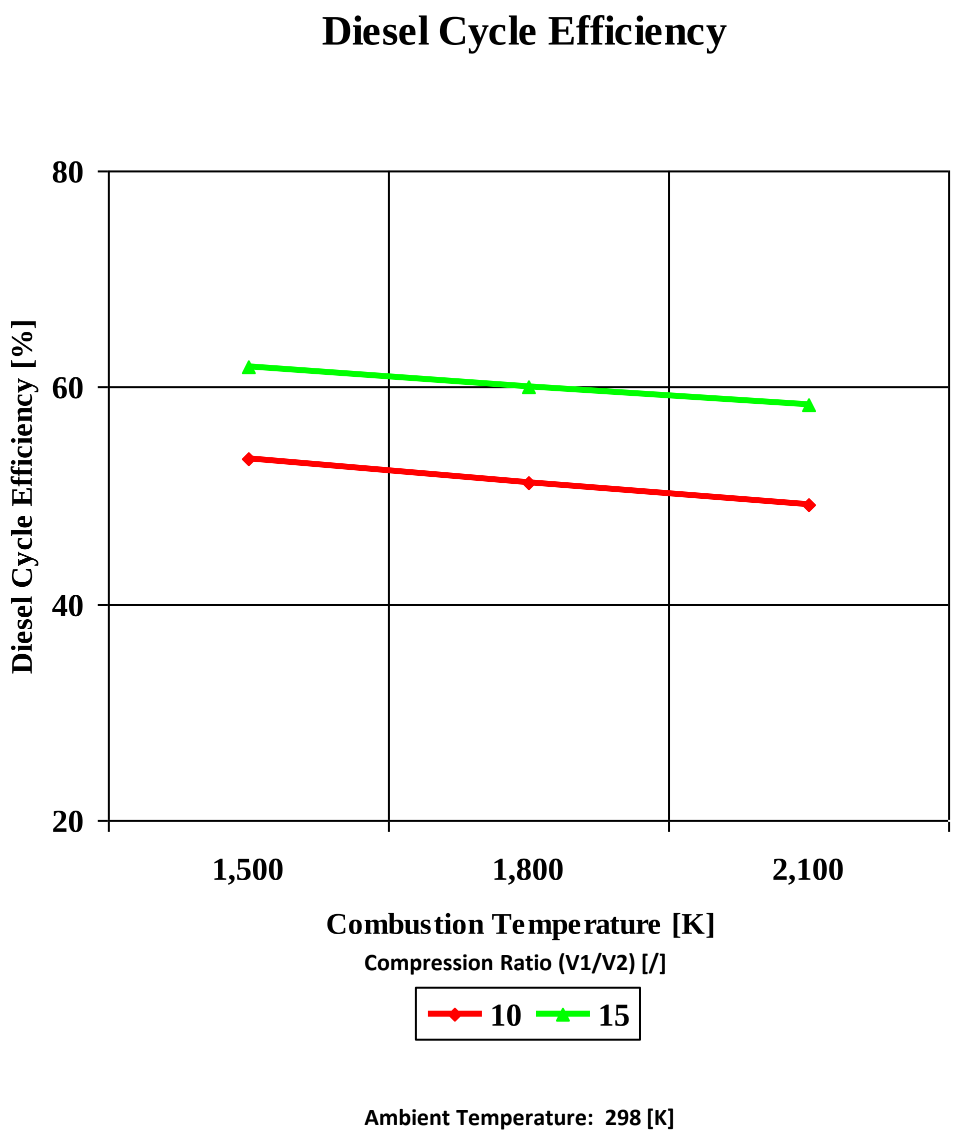
$\varepsilon = V_1/V_2$

$\varphi = V_3/V_2$

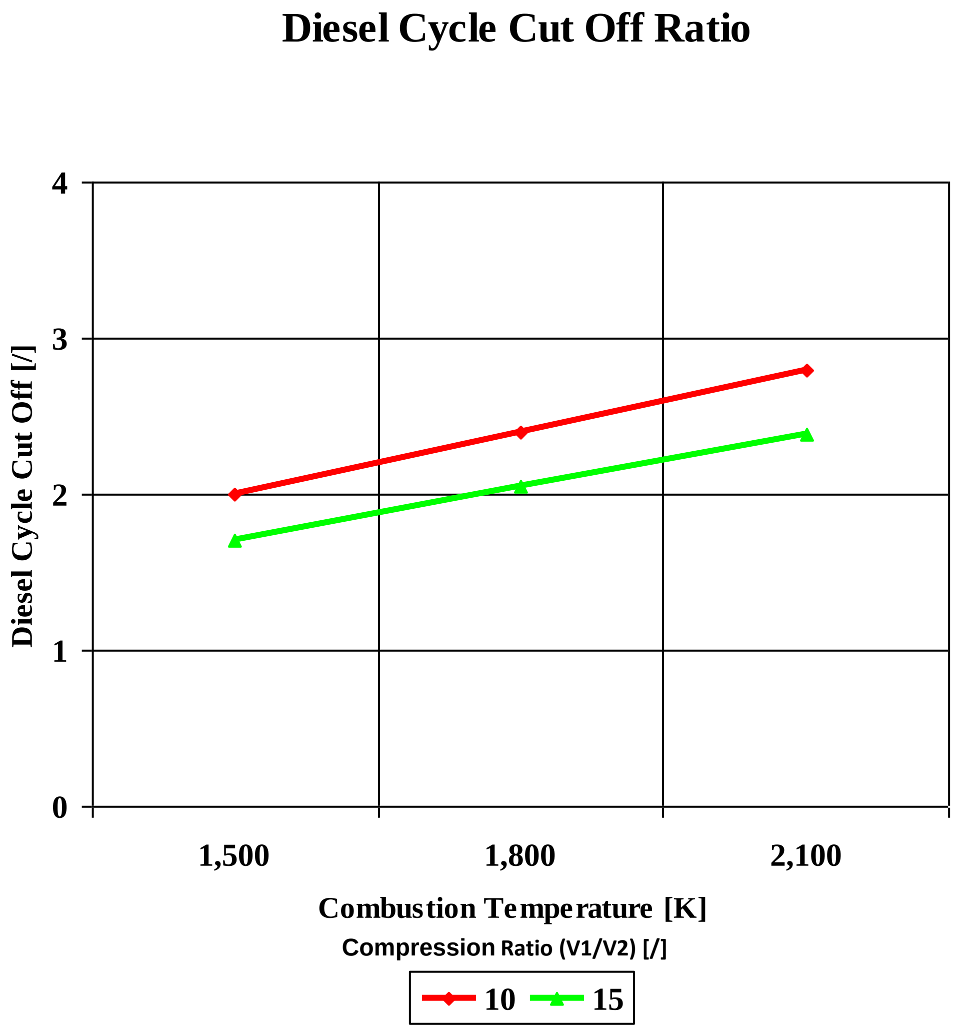
Diesel Cycle



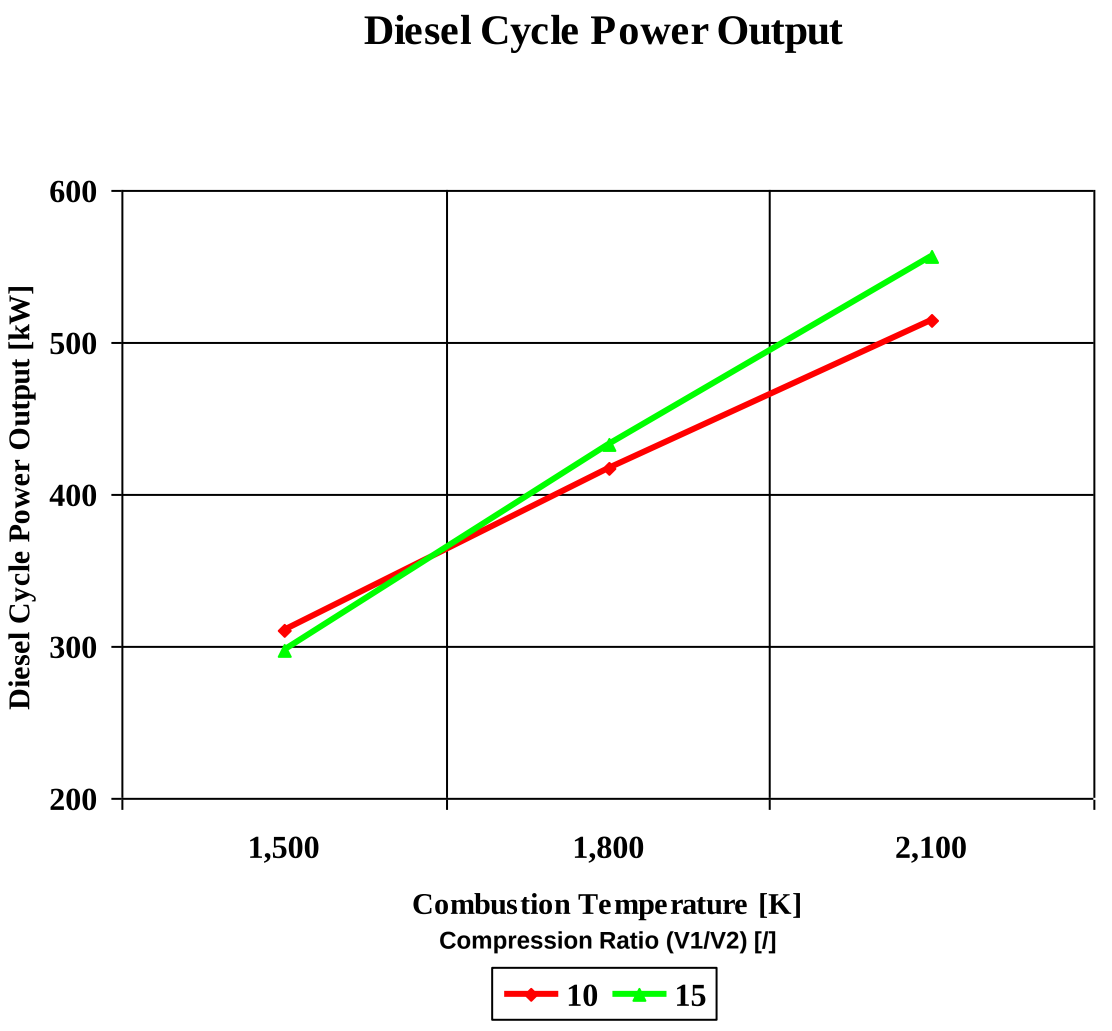
Diesel Cycle



Diesel Cycle



Diesel Cycle



Working Fluid: Air

Ambient Temperature: 298 [K] -- Number of Revolutions: 60 [1/s]

For a Given Geometry of a Four Cylinder and Four Stroke Diesel Engine